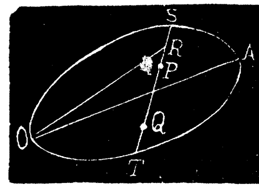
$$\triangle=\frac{2}{A^2}\int\int d\theta dp\int_0^{C-a}\int_a^{C-x} ydxdy,$$

$$=\frac{1}{A^2}\int\int d\theta dp\int_0^{C-a}(C^2-2Cx+x^2-a^2)dx,$$

$$=\frac{1}{3A^2}\int\int(C^3-3a^2C+2a^3)d\theta dp.$$



Now let the area be a circle with the origin at centre. Then $C=2\sqrt{r^2-p^2}$, when r=radius. The limits of θ are 0 and $\frac{1}{2}\pi$, doubled, of p, 0 and $\frac{1}{2}\sqrt{4r^2-a^2}$, and doubled.

$$\therefore\ \triangle=\frac{4}{3\pi^2r^4}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi\sqrt{4r^2-a^2}}\left\{8(r^2-p^2)^{\frac{3}{2}}-6a^2(r^2-p^2)^{\frac{1}{2}}+2a^3\right\}d\theta dp,$$

$$=\frac{1}{2\pi^2r^4}\left\{(a^3+2ar^2)(4r^2-a^2)^{\frac{1}{2}}+8r^2(r^2-a^2)\sin^{-1}\left(1-\frac{a^2}{4r^2}\right)^{\frac{1}{2}}\right\}\int_0^{\frac{1}{2}\pi}d\theta$$

$$=\frac{1}{4\pi}\cdot\frac{a}{r}\left(2+\frac{a^2}{r^2}\right)\left(4-\frac{a^2}{r^2}\right)^{\frac{1}{2}}+\frac{2}{\pi}\left(1-\frac{a^2}{r^2}\right)\sin^{-1}\left(1-\frac{a^2}{4r^2}\right)^{\frac{1}{2}}.$$

If $a=r$, $\triangle=\dfrac{3\sqrt{3}}{4\pi}$.

26. Proposed by J. WATSON, Middlecreek, Ohio.

Find the average area of all right-angled triangles having a given hypotenuse.

I. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Let h=the given hypotenuse, and x=the base; then will $\sqrt{(h^2-x^2)}$= the perpendicular, and the area of the triangle is $A=\frac{1}{2}x\sqrt{(h^2-x^2)}$. Hence the required average area becomes, if $\frac{1}{2}h\sqrt{2}=a$,

$$A=\int_0^a Adx\div\int_0^a dx,=\tfrac{1}{12}h^2(2\sqrt{2}-1).$$

Second Solution.

Represent the base by $h\cos\theta$, and the perpendicular by $h\sin\theta$; then we have $A=\frac{1}{2}h^2\int_0^{\frac{1}{4}\pi}\sin\theta\cos^2\theta d\theta\div h\int_0^{\frac{1}{4}\pi}\cos\theta d\theta,=\frac{1}{12}h^2(2\sqrt{2}-1)$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in Texarkana College, Texarkana, Arkansas; O. W. ANTHONY, Professor of Mathematics, New Windsor College, New Windsor, Maryland; J. F. W. SCHEFFER, A. M., Hagerstown, Maryland; and H. W. DRAUGHON, Olio, Mississippi.

Let $AC=2a$=hypotenuse of triangle, $AD=DC=DB=a$, and $\angle CDB=\theta$. $\therefore BE=a\sin\theta$.

$\therefore$ Area $=a^2 \sin\theta$. Perimeter$=2a(\cos\frac{1}{2}\theta+\sin\frac{1}{2}\theta+1$.

Let A=average area, P=average perimeter.

$$\therefore A=\frac{a^2\int_0^\pi \sin\theta d\theta}{\int_0^\pi d\theta}=\frac{2a^2}{\pi}.$$

$$P=\frac{2a\int_0^\pi(\cos\frac{1}{2}\theta+\sin\frac{1}{2}\theta+1)d\theta}{\int_0^\pi d\theta}=\frac{2a(4+\pi)}{\pi}.$$

III. Solution by **O. W. ANTHONY, Professor of Mathematics, New Windsor College, New Windsor, Maryland; P. S. BERG, Larimore, North Dakota; and H. W. DRAUGHON, Olio, Mississippi.**

Let a=the hypotenuse, and x one of the sides.

Then the area of the triangle $=\frac{1}{2}\times\sqrt{(a^2-x^2)}$ and the required average area

$$=\frac{\int_0^a \frac{1}{2}x\sqrt{(a^2-x^2)}}{\int_0^a dx}=\frac{a^2}{6}.$$

NOTE.—We have published these various solutions in order that the authors may compare their results and decide upon some definite method of solving this problem. It is our opinion that the result, $\frac{a^2}{2\pi}$, is correct; for the number of triangles is equal to the semi-circmference whose diameter is the given hypotenuse a, that is to say, the number of triangles is proportional to the locus of the vertex of the right angle and not proportional to the variable sides. But if this method of solution is adopted for this problem, it will vitiate the solutions of a great many problems in *Average* and *Probability*,—solutions that have gone in print in numerous Journals and text books.

Dr. Artemas Martin proposed this problem in the *Educational Times*, London, England, for October, 1869. The published solutions both in the *Times* and the *Reprint* give the answer $\frac{a^2}{2\pi}$. Dr. Martin says, *Mathematical Magazine*, Vol I., p. 216, "I do not regard that method [the method assuming that the vertices of the right angle are uniformly distributed on the semi-circumference of a circle whose diameter is a] as correct. The vertices of the right angle will all be situated on a semi-circumference whose diameter is a, but they will *not* be uniformly distributed on it. In order to obtain *all* the triangles, one of the legs should be made to vary uniformly from 0 to a."

He then produces a very beautiful solution without the aid of the calculus and gets as a result, $\frac{1}{6}a^2$. Then he gives another solution which is the same as III. above.

Now it seems to us that whether the triangles are uniformly distributed on the semi-circumference or not is of no concern in the solution of the problem. The question is (1), how many right triangles are there whose hypotenuses are a; and (2), what is the area of each one of these triangles? Having found the numbers answering to these questions, we divide the sum of the areas of the triangles by the number of triangles, according to the principle of *Mean Value*, and get the required result. The *sum* of the areas of the triangles is easily found by the aid of the Calculus and the number of triangles is equal to the semi-circumference of a circle whose diameter is a. This is, in our opinion, the correct solution and agrees with II. above. All of the above solutions are, doubtless, correct from the stand-points of the authors, but the stand-points of some must be wrong. As it is the object of the MONTHLY to aid in the establishment of sound principles in all departments of Mathematics, we shall be pleased to publish, in the next issue, brief notes on these solutions from various contributors. [EDITOR.]

PROBLEMS.

33. **Proposed by F. P. MATZ, M. Sc, Ph. D, Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

Find the average area of all regular polygons having a *constant* apothem.

34. **Proposed by B. F. FINKEL, A. M., Professor of Mathematics, Drury College, Springfield, Missouri.**

Two points are taken at random on the circumference of a semi-circle. Find the chance that their ordinates fall on either side of a point taken at random on the diameter.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

DIOPHANTUS' EPITAPH.

Hic Diophantus habet tumulum, qui tempora vitae
Illius mira denotat arte tibi,
Egit sextantem juvenis; languine malas
Vestire hinc coepit parte duodecima.

Septante uxori post haec sociatur, et anno
Formosus quinto nascitur inde puer.

Semissem aetatis postquam attigit ille paternae
Infelix subita morte peremptus obit.
Quatuor aestates genitor lugere superstes
Cogitur: hinc annos illius assequere.